

Heavy Quark Physics and Lattice QCD

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I review recent progress made on heavy quark physics on the lattice.

1. INTRODUCTION

Precise knowledge on B meson decay properties plays an essential role in testing the unitarity of the CKM matrix [1,2]. The experimental status relevant to the unitarity test is now promising because experimental uncertainties are already very small or expected to be reduced well in the near future [3]. On the other hand, theoretical calculations relevant to the test are still not sufficiently accurate due to the non-perturbative effect of QCD. Lattice QCD is an ideal tool to deal with this effect and should be able to reduce the current theoretical uncertainties [4].

Apart from the asymmetric e^+e^- colliders, there are two important upcoming experiments for the unitarity test. The Tevatron Run II will produce a large number of and variety of b and c -hadrons, and their basic properties such as mass and lifetime will be precisely measured [5]. What is important for us is that once B_s^0 - \bar{B}_s^0 mixing is observed the mass difference ΔM_{B_s} will be measured with a few percent accuracy. The width difference in the B_s meson system is also expected to be measured precisely. Another exciting experiment is the CLEO-c project [6]. There, charmed quarkonia, hybrid states and glueballs will be observed. In particular the leptonic and the semi-leptonic decays of D mesons are expected to be measured with a few percent level. Advancing lattice QCD calculations with a view to combining them with these precise experimental results is an urgent task in front of us.

In this review, I will focus on new and updated calculations of hadron matrix elements. The current status and progress made in spectrum calculations and formulations are not covered, although they are very important and interesting.

In particular, I will mainly discuss the lattice determination of the B^0 - \bar{B}^0 mixing amplitude and how to put a strong constraint on the poorly known CKM element $|V_{td}|$. The other important quantities such as form factors of semi-leptonic decays will be mentioned briefly. Recently several suggestions to improve the limits on accuracy of present lattice calculations have been made in methodology. I will briefly introduce some of them before summarizing my point of view on the current status.

2. B^0 - \bar{B}^0 mixing

2.1. General remarks

Within the Standard Model, the mass difference of neutral B meson system is given by

$$\Delta M_{B_q} = (\text{known factor}) \times |V_{tb}^* V_{tq}|^2 \times \frac{\langle \bar{B}_q^0 | \bar{b} \gamma_\mu (1 - \gamma_5) q \bar{b} \gamma_\mu (1 - \gamma_5) q | B_q^0 \rangle}{M_{B_q}}, \quad (1)$$

where $q=d$ or s . ΔM_{B_d} has already been measured accurately $\Delta M_{B_d} = 0.503 \pm 0.006 \text{ ps}^{-1}$ [7], while for ΔM_{B_s} only a lower bound is known ($\Delta M_{B_s} > 14.4 \text{ ps}^{-1}$ at 95 % CL [7]), but it is expected to be measured precisely at the Tevatron Run II. The hadron matrix element in (1) is usually parameterized as

$$\begin{aligned} & \langle \bar{B}_q^0 | \bar{b} \gamma_\mu (1 - \gamma_5) q \bar{b} \gamma_\mu (1 - \gamma_5) q | B_q^0 \rangle \\ &= \frac{8}{3} f_{B_q}^2 B_{B_q}(\mu_b) M_{B_q}^2, \end{aligned} \quad (2)$$

where the decay constant f_{B_q} is defined through the following matrix element,

$$\langle 0 | \bar{b} \gamma_\mu (1 - \gamma_5) q | B_q^0 \rangle = i f_{B_q} p_\mu. \quad (3)$$

Since the four-quark operator receives renormalization, B_B depends on renormalization scheme

year	f_{B_d} [MeV]	\hat{B}_{B_d}
'97 Onogi	163(12)	
'98 Draper	165(20)	1.32(6)(12)
'99 Hashimoto	170(20)	1.23(23)
'00 Bernard	175(20)	1.30(12)
'01 Ryan	173(23)	1.30(12)

Table 1

The quenched f_{B_d} and \hat{B}_{B_d} quoted in the recent reviews.

and scale. In the literature, the renormalization scale independent B parameter is often used, which is defined by

$$\hat{B}_{B_q} = [\alpha_s(\mu_b)]^{-\frac{6}{23}} \left[1 + \frac{\alpha_s(\mu_b)}{4\pi} J_5 \right] B_{B_q}(\mu_b), \quad (4)$$

where the full expression of J_{n_f} is found in [8]. Throughout this review, I quote \hat{B}_{B_q} rather than the scale dependent one $B_{B_q}(\mu)$.

In the following, I summarize the current status of the quenched and unquenched calculations of f_{B_q} and \hat{B}_{B_q} .

2.2. Quenched f_{B_q} and \hat{B}_{B_q}

According to the recent Lattice conference reviews [9–13], the quenched results for f_{B_d} and \hat{B}_{B_d} have been stable over several years as shown in Table 1. This year JLQCD [14] updated their quenched calculation, which is performed at $\beta=6.0$ using the non-perturbatively $O(a)$ improved Wilson light quark and NRQCD for heavy quarks. They find $f_{B_d} = 158(3)(10)$ MeV, $f_{B_s}/f_{B_d} = 1.16(1)(^{+0.04}_{-0.00})$, $\hat{B}_{B_d} = 1.29(5)(7)$ and $\hat{B}_{B_s}/\hat{B}_{B_d} = 1.020(24)(15)(^{+4}_{-0})$, where the first error is statistical, the second systematic and the third comes from the uncertainty of m_s .

Figure 1 summarizes recent results for quenched f_{B_d} [15–25] including the JLQCD's updated result [14]. The results are grouped into three categories, “FNAL” [26], “NRQCD” [27] and “NPSW”, depending on the formulation for heavy quark¹. We find an agreement within about 10% among the results obtained with the three different actions. Since the new result by

¹For a brief summary on the actions used, for example, see [13].

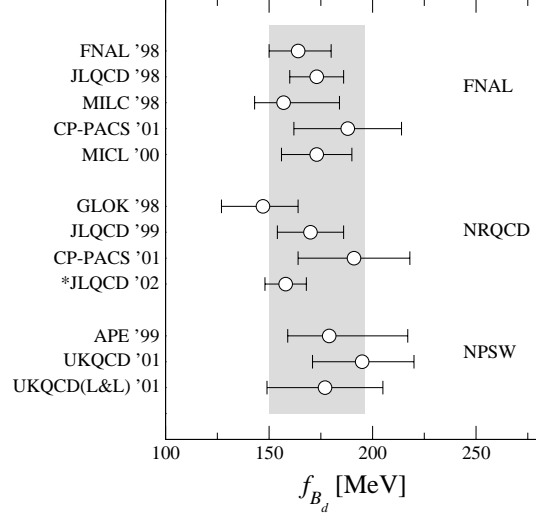


Figure 1. Summary plot of quenched f_{B_d} . Shaded band represents the current average given by Ryan [13].

JLQCD is consistent with the previous average given by Ryan [13] (shaded band in the figure), I keep her values as the summary as of this conference.

$$\begin{aligned}
 f_{B_d}^{(N_f=0)} &= 173(23)\text{MeV}, \\
 f_{B_s}^{(N_f=0)} &= 200(20)\text{MeV}, \\
 f_{D_s}^{(N_f=0)} &= 203(14)\text{MeV}, \\
 f_{D_s}^{(N_f=0)} &= 230(14)\text{MeV}.
 \end{aligned} \quad (5)$$

These values are helpful in calibrating new formulation of heavy quarks.

As for \hat{B}_{B_q} , only a couple of calculations are available. Figure 2 shows the $1/M_{P_d}$ dependence of \hat{B}_{B_d} obtained by APE [28] with the relativistic heavy quark (NPSW) and by JLQCD [14] with NRQCD for heavy quark. UKQCD has also used the relativistic heavy quark [25], but since their result agrees with the APE result very well it is suppressed in the figure for simplicity. I also plot the result calculated in the static limit (HQET) by Gimenez and Martinelli [29]².

The $1/M_{P_d}$ dependence of \hat{B}_{B_d} seems inconsistent between two formulations of heavy quark,

²Later their result was corrected in [30].

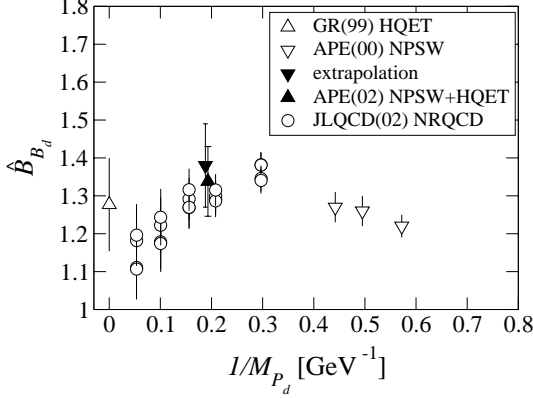


Figure 2. Comparison of $1/M_{P_d}$ dependence of \hat{B}_{B_d} .

but the results agree within error at the physical point $1/M_{B_d}=0.19 \text{ GeV}^{-1}$. A possible reason for this disagreement is systematic errors, which are different for different formulations; the relativistic action has sizable $O(a^2 m^2)$ errors and the NRQCD approach contains large $O(\alpha_s^2)$ errors in the matching. In [31], the results with the relativistic approach is combined with the one obtained in the static limit and interpolated to the physical B_d meson mass (filled triangle up). It is interesting that the result with the combined analysis is slightly lower than that with extrapolation (filled triangle down) and the consistency with the NRQCD becomes better.

SPQcdR has started a new calculation at $\beta=6.45$ [32] with the RI-MOM scheme [33] for renormalization. The strategy is similar to what APE adopted, namely relying on the extrapolation in heavy quark mass from around c quark region to the physical b quark mass. However, due to the use of high β value, the distance to the physical B meson mass becomes smaller. The motivation of this work is to test if the $O(a^2)$ error is under control. While the numerical results are still very preliminary, no significant change has been observed compared to the APE results at $\beta=6.2$ [28]. This suggests that the discretization error, which is most serious in this extrapolation method, is under control.

Figure 3 summarizes the quenched calculations

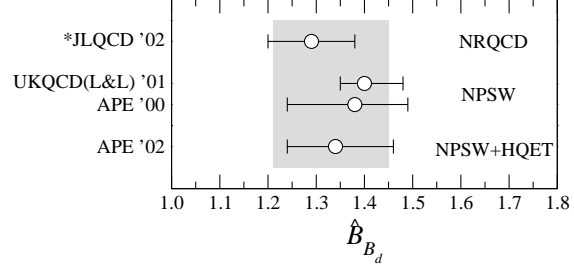


Figure 3. Summary of quenched \hat{B}_B . Shaded band represents the current average.

of \hat{B}_{B_d} . At this moment, I quote the following conservative value as a current estimate,

$$\hat{B}_{B_d}^{(N_f=0)} = 1.33(12), \quad (6)$$

which is indicated in the figure by shade.

2.3. Unquenched f_{B_q}

Several large scale unquenched calculations of f_B were carried out a few years ago, and the recent reviews reported 10–20 % increase compared to the quenched ones [10–13]. I wish to discuss that this conclusion needs a reexamination because of possible effects of chiral logarithms expected from chiral perturbation theory.

Incorporating heavy quarks into chiral perturbation theory *via* heavy quark symmetry was first proposed by Wise [34] and Burdman and Donoghue [35]. Since then several authors have developed this idea [36]. Recently Booth [37] and Sharpe and Zhang [38] extended the idea to (partially) quenched chiral perturbation theory ((P)QChPT) to make it applicable to actual lattice calculations.

In the $N_f=2$ unquenched case, where u and d quarks are treated dynamically, PQChPT predicts the non-analytic term in f_{B_d} to be [38]

$$-\frac{3}{4}(1+3g^2) \frac{m_\pi^2}{(4\pi f)^2} \ln(m_\pi^2/\Lambda^2), \quad (7)$$

where Λ is the cutoff of the theory and $f=f_\pi$ at the order considered. While f_π has a similar expression for its non-analytic term, in this case the $B^* B \pi$ coupling g , which is only poorly constrained in $g=0.3 \sim 0.6$ [36], appears because the

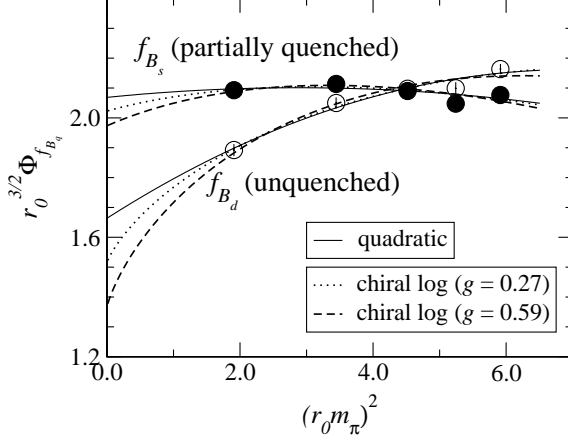


Figure 4. Chiral extrapolation of $\Phi_{f_{B_q}}$ given by [39]. Both axis are normalized by Sommer's scale r_0 [40].

mass difference in B and B^* mesons is smaller than the pion excitation. For f_{B_s} , the expression of the non-analytic term takes a different form. In the $N_f=2$ partially quenched case, where u and d quarks are treated dynamically as before and s appears only as a valence quark, the prediction for the non-analytic term becomes

$$-(1 + 3g^2) \frac{m_{s\bar{q}}^2}{(4\pi f)^2} \ln(m_{s\bar{q}}^2/\Lambda^2), \quad (8)$$

where $m_{s\bar{q}}^2 = (m_{s\bar{s}}^2 + m_\pi^2)/2$. We observe that while for the chiral behavior of f_{B_d} (7) indicates significant slope as m_π^2 vanishes, (8) suggests a milder behavior for f_{B_s} because $m_{s\bar{q}}^2$ becomes a constant when $m_{s\bar{s}}^2 \gg m_\pi^2$. Lattice data are supposed to show these logarithmic behaviors.

JLQCD has accumulated twice as many statistics as they had last year in $N_f=2$ QCD, and updated their analysis of the decay constant [39]. The simulation is performed at $\beta=5.2$ ($a \sim 0.09$ fm) with the NRQCD action for heavy quark and the non-perturbatively $O(a)$ -improved Wilson quark action for both dynamical and valence light quarks, which range from $0.5 m_s$ to $1.5 m_s$. The chiral extrapolation of $\Phi_{f_{B_q}} = f_{B_q} \sqrt{M_{B_q}}$ is shown in Fig. 4. They attempt to fit $\Phi_{f_{B_d}}$ and $\Phi_{f_{B_s}}$ with (7) and (8), respectively, as well as with a conventional quadratic fit. They find that f_{B_d}

depends on the fit form and the $B^*B\pi$ coupling g significantly, while f_{B_s} does not, in agreement with the observation made above. This means that f_{B_s}/f_{B_d} has large uncertainties as well. From this analysis, they give $f_{B_s}/f_{B_d}=1.24-1.38$ as a preliminary result.

Recently, Kronfeld and Ryan [41] pointed out the similar enhancement of the ratio by taking into account the predicted chiral log and reconsidering the chiral extrapolation. Their estimate is $f_{B_s}/f_{B_d}=1.32(8)$ and $B_{B_s}/B_{B_d}=1.00(2)$. Both analyses suggest that the ratio can be significantly larger than the previous world average $f_{B_s}/f_{B_d}=1.16(5)$ [13].

Here let me summarize the $N_f=2$ unquenched calculations of f_{B_q} . At present, consistency between lattice data and ChPT is not clear because the value of g is still unknown. In addition, the $O(1/M)$ contribution to the log term is not known well [42]. In this review, the central value of decay constants is taken from the previous ones [13], which can be considered as those obtained with a quadratic fit in the chiral extrapolation. The uncertainty for f_{B_d} associated with the chiral logarithm is estimated following JLQCD while it is neglected for f_{B_s} . According to their analysis,

$$(f_{B_d}^{\text{qua}} - f_{B_d}^{\text{ChPT}})/f_{B_d}^{\text{qua}} = 0.17, \quad (9)$$

where $f_{B_d}^{\text{qua}}$ is obtained by quadratic form (as usual) and $f_{B_d}^{\text{ChPT}}$ includes the effect of chiral logarithm. In obtaining $f_{B_d}^{\text{ChPT}}$, they take the upper bound $g=0.6$, for which the effect becomes maximum. Taking this uncertainty into account, I quote

$$f_{B_d}^{(N_f=2)} = 198(30)_{-34}^{+0} \text{ MeV}, \quad (10)$$

$$f_{B_s}^{(N_f=2)} = 230(30) \text{ MeV}, \quad (11)$$

$$f_{B_s}^{(N_f=2)}/f_{B_d}^{(N_f=2)} = 1.16(5)_{-0}^{+24}, \quad (12)$$

as my conservative estimates.

The physical prediction of $f_{B_{d,s}}$ requires three-flavor dynamical simulations. MILC has started such an attempt [43]. They employ a highly improved gauge and staggered quark actions for gauge configurations and the tadpole-improved clover action for valence quarks, where FNAL formalism is applied for heavy quarks. The simulation is performed for two lattice spacings $a \sim 0.13$

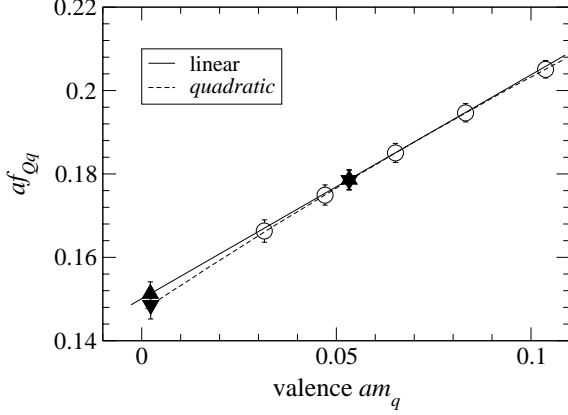


Figure 5. Chiral extrapolation of af_{Qq} in valence quark mass given by [43].

and 0.09 fm. To keep the lattice spacing constant, β is adjusted as dynamical quark mass is varied. Since Z_A is not available yet, they focus on ratios. Chiral extrapolations are made in two steps. First the valence light quark is extrapolated (interpolated) to the physical up/down (strange) quark mass at each dynamical quark mass. Then dynamical quark mass is extrapolated to up/down. The third flavor of dynamical quark is tuned to m_s in advance.

Figure 5 shows the chiral extrapolation of af_{Qq} in valence quark mass. They attempt to fit the data by two types of fit forms, linear (solid line) and quadratic (dotted line) functions, and find no significant deviation between them. It is interesting to see whether this linear behavior is consistent with $N_f=3$ PQChPT. But such a test is complicated because of flavor breaking effects in pion masses for the staggered quark action [44].

The chiral extrapolations of f_{B_d} and f_{B_s}/f_{B_d} in dynamical quark mass are shown in Figs. 6 and 7, respectively. In 6, where the scale is set by m_ρ , a negative slope is seen. But once one sets the scale by r_1 [45], the slope becomes unclear, namely f_{B_d} appears to be independent on m_{dyn} . In 7, they find that the ratio is independent both of N_f and m_{dyn} , and quote $f_{B_s}/f_{B_d} = 1.18(1)_{-1}^{+4}$ (preliminary). It is very interesting to check if the whole above observations are consistent with ChPT.

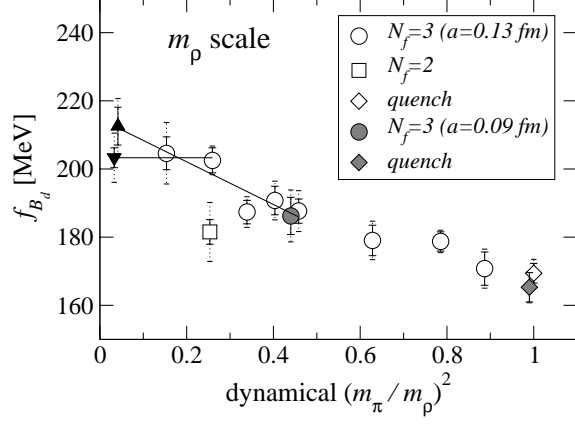


Figure 6. Chiral extrapolation of f_{B_d} in dynamical quark mass given by [43].

2.4. unquenched \hat{B}_{B_q}

This year no new or updated calculation on the unquenched B -parameters was presented. Last year JLQCD reported that the fit form dependence is relatively mild for \hat{B}_{B_q} in their $N_f = 2$ unquenched simulation [46]. This is consistent with the PQChPT prediction that the chiral logarithm term for \hat{B}_{B_d} is given by [38] as

$$-\frac{1}{2}(1-3g^2)\frac{m_\pi^2}{(4\pi f)^2}\ln(m_\pi^2/\Lambda^2), \quad (13)$$

and hence suppressed compared with the case of f_{B_d} in (7), and that chiral logarithm is altogether absent for \hat{B}_{B_s} . JLQCD also reported that quenching effects are small from a comparison with the quenched \hat{B}_{B_q} .

The ratio $\hat{B}_{B_s}/\hat{B}_{B_d}$ is also stable against the number of dynamical flavor, the heavy quark action and the fit form in the chiral extrapolation, and the recent results are compared in Fig. 8, where only the data shown in top is unquenched results.

Since at present there is only one unquenched calculation by JLQCD, I take their values as current estimates, and quote

$$\hat{B}_{B_d}^{(N_f=2)} = 1.34(13), \quad (14)$$

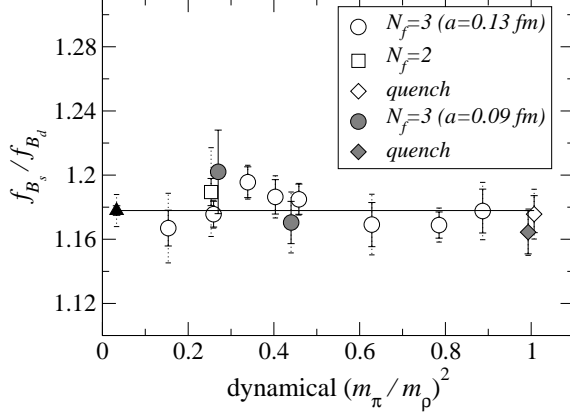


Figure 7. Chiral extrapolation of f_{B_s}/f_{B_d} in dynamical quark mass given by [43].

$$\frac{\hat{B}_{B_s}^{(N_f=2)}}{\hat{B}_{B_d}^{(N_f=2)}} = 0.999(24). \quad (15)$$

It should be noted that the ratio is already very precise.

2.5. SU(3) flavor breaking ratio

The Tevatron Run II experiment is expected to yield a precise value for the mass difference of B_s meson system, ΔM_s , in the near future [5]. Once ΔM_s is determined precisely, assuming the three generation unitarity ($|V_{ts}| \approx |V_{cb}|$) we can put a strong constraint on the poorly known CKM matrix element $|V_{td}|$ through the SU(3) flavor breaking ratio of the hadron matrix element,

$$\frac{\Delta M_{B_s}}{\Delta M_{B_d}} = \frac{|V_{ts}|^2}{|V_{td}|^2} \frac{M_{B_s}}{M_{B_d}} \xi^2, \quad (16)$$

where $\xi = (f_{B_s} \sqrt{\hat{B}_{B_s}})/(f_{B_d} \sqrt{\hat{B}_{B_d}})$.

Until recently it has been expected that ξ can be easily determined within a few percent accuracy by lattice QCD. However, the presence of chiral logarithm makes it more difficult. As we discussed in Sec. 2.3, f_{B_s}/f_{B_d} can still have a large systematic uncertainty while the ratio of \hat{B}_{B_q} does not. Following the values given in (12) and (15), I quote as my personal estimate,

$$\xi^{(N_f=2)} = 1.16(6)_{(-0)}^{(+24)}. \quad (17)$$

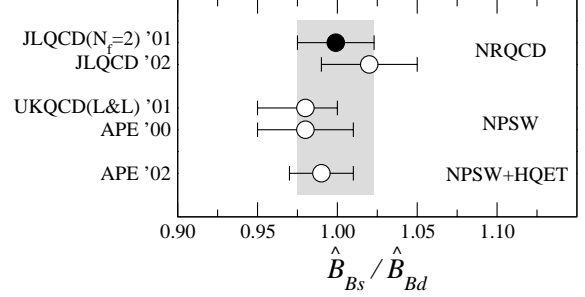


Figure 8. Comparison of quenched and unquenched $\hat{B}_{B_s}/\hat{B}_{B_d}$. Only the top one is obtained from an unquenched simulation. The central value I quote is indicated by shade.

Kronfeld and Ryan also gave a similar value ($\xi=1.32(10)$) in their analysis [41].

I now discuss that a precise determination of $|V_{td}|$ is still possible if one considers the Grinstein ratio of decay constants [47] defined by

$$R_1 = (f_{B_s}/f_{B_d})/(f_{D_s}/f_{D_d}). \quad (18)$$

Rewriting (16) in terms of R_1 , one obtains

$$\left(\frac{\Delta M_{B_s}}{\Delta M_{B_d}} \right)_{\text{RunII}} = \frac{|V_{ts}|^2}{|V_{td}|^2} \left(\frac{M_{B_s}}{M_{B_d}} \right)_{\text{known}} \times \left(\frac{f_{D_s}^2}{f_{D_d}^2} \right)_{\text{CLEO-c}} \left(\frac{\hat{B}_{B_s}}{\hat{B}_{B_d}} R_1^2 \right)_{\text{Lattice}}. \quad (19)$$

As indicated in the subscript, the ratio f_{D_s}/f_{D_d} is expected to be measured in CLEO-c, and $\hat{B}_{B_s}/\hat{B}_{B_d}$ can be determined by lattice QCD precisely. Therefore R_1 is the only remaining quantity to be fixed.

We expect this task is achievable in lattice QCD since variations under chiral and heavy quark expansions will both cancel out in the ratio to a large extent, leaving only a small deviation from unity.

This year JLQCD presented their preliminary result of R_1 [48]. The simulation is carried out on $N_f=2$ dynamical configurations at $\beta=5.2$. In order to handle the c quark, FNAL formalism is applied for heavy quarks. Figure 9 shows the chiral behavior of $R_1 \times \sqrt{(M_{B_s}/M_{B_d})/(M_{D_s}/M_{D_d})}$. Several fit forms are attempted as shown in the

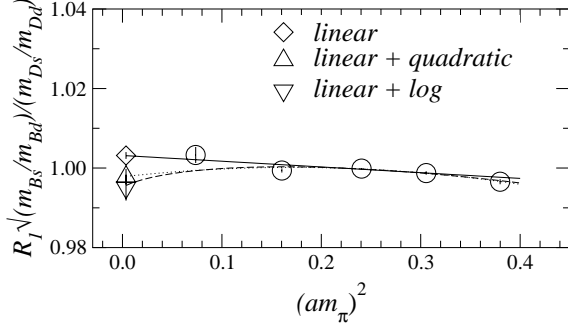


Figure 9. Chiral extrapolation of $R_1 \times \sqrt{(M_{B_s}/M_{B_d})/(M_{D_s}/M_{D_d})}$ given by [48].

figure. They found that the result is relatively insensitive to the fit form and its dependence is, at most, about 1%. Their preliminary result is $R_1 = 1.018(06)(10)$, where the first error is statistical and the second represents the systematic uncertainty.

In Fig. 10, I gather the Grinstein ratio obtained by previous calculations. Only FNAL'98 [15] and JLQCD'02(clover) [48] explicitly calculated R_1 . For other results, I only show their central values. It can be seen that this ratio is very stable and there is only a negligible fluctuation. At present, I quote

$$R_1^{(N_f=2)} = 1.02(2). \quad (20)$$

From a comparison with quenched and unquenched ($N_f=2$) results, it is unlikely that the central value and its accuracy change drastically when one goes to $N_f=3$.

As we have seen, once ΔM_{B_s} , f_{D_s} and f_{D_d} are measured in the forthcoming experiments, we can test the unitarity at a few percent level through $|V_{td}|$. To gain more confidence, the current values of R_1 and $\hat{B}_{B_s}/\hat{B}_{B_d}$ should be confirmed by several groups.

3. Form factors

3.1. heavy to heavy transition

In order to test the CKM unitarity, a precise determination of $|V_{cb}|$ is indispensable, as it sets the normalization of the parameters (ρ, η) . To

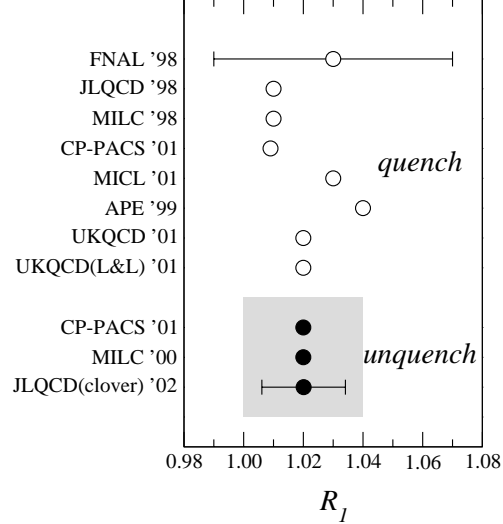


Figure 10. Summary of the Grinstein ratio.

this end, $\bar{B} \rightarrow D^* l \bar{\nu}_l$ is most promising. The differential decay rate is given by

$$\frac{d\Gamma}{d\omega} = (\text{known factor}) |V_{cb}|^2 |\mathcal{F}_{B \rightarrow D^*}(\omega)|^2, \quad (21)$$

where $\omega = v'_{D^*} \cdot v_B$. At zero recoil ($\omega = 1$), $\mathcal{F}_{B \rightarrow D^*}(1) = h_{A_1}(1)$, where $h_{A_1}(1)$ is given by

$$\langle D^*(v) | \bar{c} \gamma_\mu \gamma_5 b | \bar{B}(v) \rangle = i \sqrt{2m_B 2m_{D^*}} \epsilon'_\mu h_{A_1}(1). \quad (22)$$

Last year Fermilab group demonstrated a precise determination of $\mathcal{F}_{B \rightarrow D^*}(1)$ by a full use of heavy quark symmetry and obtained $\mathcal{F}_{B \rightarrow D^*}(1) = 0.913^{+0.0238+0.0171}_{-0.0173-0.0302}$ in the quenched approximation [49]. In my opinion the lattice method for calculation of this decay mode has been established. The extension to unquenched calculations remains to be done. It is worth noting that chiral logarithm terms would not affect this accuracy because they are suppressed by $(1/M)^2$.

At this conference a new calculation of $\Lambda_b \rightarrow \Lambda_c l \nu$ was presented by Tamhankar [50]. This process is experimentally challenging, but gives an independent determination of $|V_{cb}|$. They use the quenched $20^3 \times 64$ lattice with $a^{-1} = 1.32$ GeV. The actions used are $O(a^2, \alpha_s a^2)$ improved gauge and tadpole improved clover and FNAL approach is taken for b and c quarks. The chiral limit is not taken at present. Instead, they investigate the

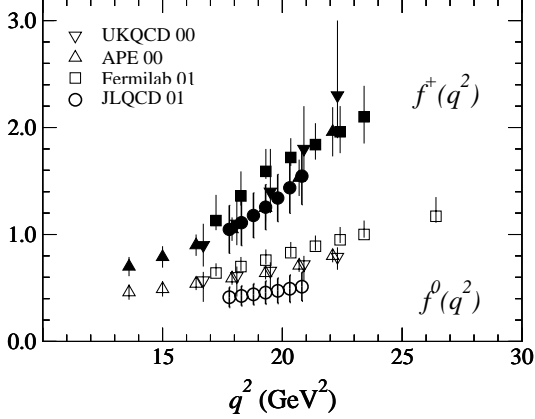


Figure 11. The current status of quenched calculations of the $B \rightarrow \pi l \nu$ form factors.

initial heavy quark mass (m_b) dependence with fixed m_c and m_{light} . No clear mass dependence has been observed in their preliminary analysis.

3.2. heavy to light transition

Once $|V_{td}|/|V_{cb}|$ and angle $\phi_1(\beta)$ are determined precisely enough, the location of the apex is essentially fixed in the ρ - η plane. Then the next step is to test the consistency of the CKM mechanism, for example, by measuring other CKM elements such as $|V_{ub}|$. $B \rightarrow \pi l \nu$ is one of the simplest choice for such a purpose. The definition of form factors, f^+ and f^0 , is given by

$$\begin{aligned} & \langle \pi(p') | \bar{q} \gamma_\mu b | B(p) \rangle \\ &= (p_\mu + p'_\mu - \frac{m_B^2 - m_\pi^2}{q^2} q_\mu) f^+(q^2) \\ &+ \frac{m_B^2 - m_\pi^2}{q^2} q_\mu f^0(q^2), \end{aligned} \quad (23)$$

where $q = p - p'$. Figure 11 [51] shows the quenched calculations made by four major groups [52–55]. Data from different groups agree with each other, but only within a large uncertainty ($\sim 20\%$). Unquenched simulations remain to be done. Moreover the issue of chiral logarithm could be a significant source of uncertainty in this case. More studies are needed before finalizing the form factor calculation.

This year SPQcdR made two contributions on

the heavy to light vector meson semi-leptonic decays, $B \rightarrow \rho l \nu$ ($D \rightarrow K^* l \nu$) [56] and $B \rightarrow K^* \gamma$ [57]. Both are performed at $\beta=6.2$ and 6.45 with non-perturbatively $O(a)$ -improved Wilson action for light and heavy quarks, and the currents are also improved non-perturbatively. The form factors of these decays obtained around c quark mass is extrapolated to the physical b quark mass by using the HQET scaling laws for a fixed value of $v \cdot p'$ or $q^2=0$. According to their preliminary results, no scaling violation has been seen for both matrix elements, but the statistical uncertainty is still significant. At present, chiral extrapolations are carried out by assuming the form factors to be a linear function of light vector meson mass. But in the future a guiding principle will be necessary to make a extrapolation more reliable.

3.3. Determination of $g_{D^* D \pi}$ coupling

The determination of the coupling constant g defined in ChPT, or equivalently $g_{P^* P \pi}$ given by

$$\begin{aligned} & \langle P(p) \pi(q) | P^*(p', \lambda) \rangle \\ &= -g_{P^* P \pi}(q^2) q \cdot \epsilon^\lambda(p') (2\pi)^4 \delta(p' - p - q), \end{aligned} \quad (24)$$

$$g_{P^* P \pi} = \lim_{q^2 \rightarrow m_\pi^2} g_{P^* P \pi}(q^2). \quad (25)$$

is now extremely important because this coupling plays a crucial role in the chiral extrapolation of decay constant and in the normalization of one of the $P \rightarrow \pi$ form factor $f^+(q^2)$ at $q^2=q_{\text{max}}^2$. $g_{P^* P \pi}$ is directly calculable on the lattice, and more interestingly $g_{D^* D \pi}$ has been measured in CLEO through the $D^{*+} \rightarrow D^{0(+)} \pi^{+(0)}$ decays [58] while $B^* \rightarrow B \pi$ is prohibited kinematically. The relation between g and $g_{P^* P \pi}$ is given by ChPT as $g_{P^* P \pi} = 2 g \sqrt{m_{P^*} m_P} / f_\pi$ up to $O(1/M)$ and $O(m_\pi^2)$ corrections. Using an LSZ reduction of the pion and the PCAC relation, the calculation of above matrix element reduces to that of

$$\langle P(p) | \bar{q} \gamma_\mu \gamma_5 q | P^*(p+q) \rangle, \quad (26)$$

namely, to the calculation of the form factors describing the above matrix element.

The first exploratory study to determine $g_{P^* P \pi}$ on the lattice was made in the static limit by UKQCD [59]. This year the determination of $g_{D^* D \pi}$ at physical D meson mass was presented by Herdoiza in the quenched approximation [60].

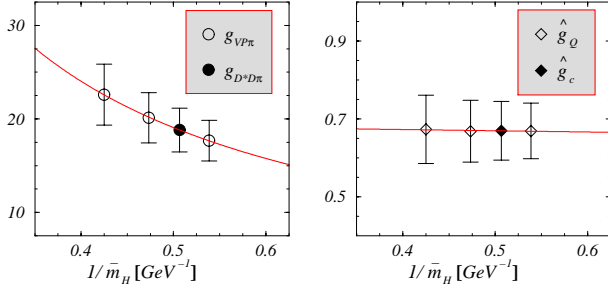


Figure 12. The $1/M_P$ dependence of $g_{D^*D\pi}$ and g in the chiral limit.

Figure 12 shows the $1/M_P$ dependence of $g_{D^*D\pi}$ and g in the chiral limit. It is found that the $1/M_P$ dependence of the former is completely canceled by the factor $\sqrt{m_P^* m_P}$. After interpolation to the physical D meson mass, they obtain $g_{D^*D\pi}=18.8(2.3)_{(-2.0)}^{(+1.1)}$, which agrees with $g_{D^*D\pi}=17.9(0.3)(1.9)$ measured by CLEO [58].

It seems that the first step toward the determination has been completed. The next step is the unquenched simulation with special care to the chiral extrapolation.

4. Improvements in methodology

4.1. KS as light quark

We have seen that the issue associated with chiral logarithm is present almost everywhere. One possible course to simulate light sea quark masses is to employ staggered fermions.

At the last conference, Wingate *et al.* performed an exploratory study of B mesons with the NRQCD and staggered actions on a coarse lattice ($a^{-1}=0.8$ GeV), and pointed out a difficulty associated with contaminations from unphysical modes [61]. This year they repeated the similar calculations on slightly finer lattices using $N_f=0$ ($a^{-1}=1$ GeV) and $N_f=3$ ($a^{-1}=1.3$ GeV) configurations [62]. They found that such unphysical modes can be essentially removed by the use of finer lattice. They also observed that the correlation functions with both parities is well separated by applying suitable fit forms and the constrained curve fitting [63].

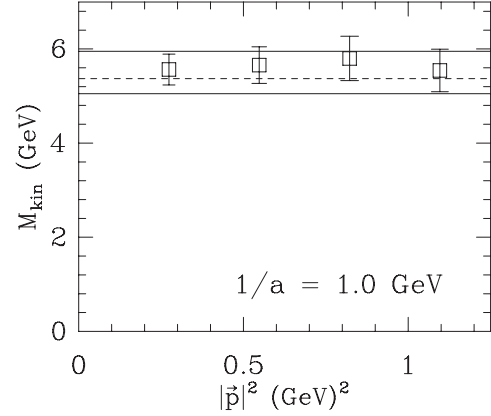


Figure 13. Comparison of the mass obtained from dispersion relation (open square) with that using perturbation theory (dashed line). The solid lines indicate the perturbative uncertainty in the latter.

They calculated B_s meson spectrum, and made a comparison of M_{B_s} obtained from dispersion relation with that using perturbation theory as shown in Fig. 13. These two definitions of mass agree quite well. They also obtained $f_{B_s}^{(N_f=0)}=225\pm 9(\text{stat.})\pm 20(\text{pert.})$ MeV in the quenched approximation and this value is reasonably consistent with the current quenched world average $f_{B_s}^{(N_f=0)}=200(20)$ MeV. More importantly they conclude that the unquenching dramatically improves the hyperfine splitting $M_{B_s^*} - M_{B_s}$ and results in 42.5 ± 3.7 MeV. These observations indicate that there is no practical problem in the combination of NRQCD and staggered fermions.

4.2. Anisotropic lattice

The motivation for the use of anisotropic lattices in heavy-light system is to obtain clean signals in the form factor calculations. The method has been applied to the $B \rightarrow \pi$ semileptonic decay by Shigemitsu *et al.* [64]. The simulation is performed on a $12^3 \times 48$ anisotropic quenched lattice with $a_s/a_t=2.71$ ($1/a_s=1.2$ GeV) using the tadpole-improved Symanzik gluon action, NRQCD for heavy quarks and D234 action for light quarks. They apply the constrained fits

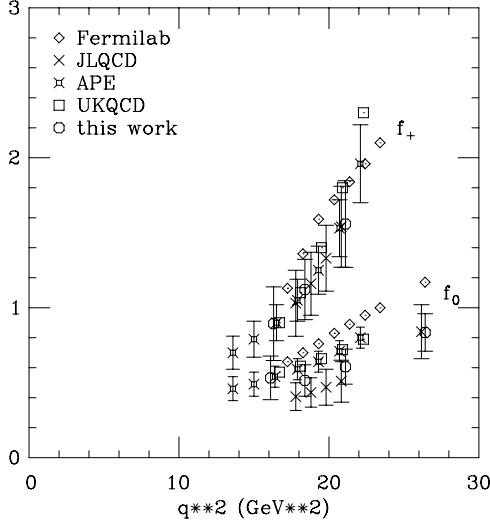


Figure 14. Comparison of form factors obtained with iso- and aniso-tropic lattice given by [64].

[63] to the three-point functions to obtain the form factors shown in Fig. 14. Their results agree well with those with isotropic lattices. In spite of a relatively small number of configurations (199), the statistical error in this work is comparable or even smaller than those of JLQCD, which used NRQCD heavy quarks and was obtained with more than 1,000 configurations [55].

4.3. Step scaling method

At this conference a new method to calculate f_B in non-perturbative accuracy was presented by Petronzio [65], based on a non-perturbative recursive finite size technique. The explicit calculation of f_B is made in the Schrödinger functional setup with several quenched lattices. Most of relevant renormalization constants are obtained non-perturbatively. My understanding of this method is as follows. Let us start from the following identity,

$$f_B(12 \text{ GeV})|_{1.6} = f_B(12 \text{ GeV})|_{0.4} \times \frac{f_B(12 \text{ GeV})|_{0.8}}{f_B(12 \text{ GeV})|_{0.4}} \times \frac{f_B(12 \text{ GeV})|_{1.6}}{f_B(12 \text{ GeV})|_{0.8}}, \quad (27)$$

where $f_B(12 \text{ GeV})|_L$ denotes f_B measured on the lattice with $a^{-1}=12 \text{ GeV}$ and the physical volume

$L \text{ fm}$. The first factor $f_B(12 \text{ GeV})|_{0.4}$ is interpreted as f_B on a finite volume and the remaining factors are considered as corrections to obtain f_B with large volume. It should be noted that with this lattice spacing the b quark can be suitably treated by the relativistic formalism and that the above equation holds even if the smallest lattice is in the deconfined phase. Since it is difficult to realize the lattice simulation with both $a^{-1}=12 \text{ GeV}$ and $L=1.6 \text{ fm}$ simultaneously, consider the following,

$$f_B(12 \text{ GeV})|_{1.6} \approx f_B(12 \text{ GeV})|_{0.4} \times \frac{f_B(6 \text{ GeV})|_{0.8}}{f_B(6 \text{ GeV})|_{0.4}} \times \frac{f_B(3 \text{ GeV})|_{1.6}}{f_B(3 \text{ GeV})|_{0.8}}, \quad (28)$$

which is obtained by replacing the second and third factors by those obtained on coarser lattices but with the same physical volume. They performed above calculation and obtained $f_B=170(11)(5)(22) \text{ MeV}$ and $f_{B_s}=192(9)(5)(24)$, where the first error is statistical and the second and third are systematic. These values agree well with the current quenched estimates (5).

The questions are how (28) is justified, what is f_B in the deconfined phase, and what is the advantage of this method compared to the calculation on a single lattice with $a^{-1}=3 \text{ GeV}$ and 1.6 fm . The possible systematic uncertainties are still unclear to me. The same calculation in the static limit would be helpful to make this clear.

5. Summary

We are now going into an exciting era because Belle, BaBar, Tevatron Run II and CLEO-c will be giving us a wealth of precision data for B and D mesons soon.

One of the CKM matrix elements that these experiments should pin down is $|V_{td}|$. An important point realized recently is that the amplitude of $B^0-\bar{B}^0$ mixing necessary to convert the experimental mass difference to this matrix element suffers from a sizable uncertainty of 10–20% due to chiral logarithms in the chiral extrapolation. However, one can avoid this problem if one introduces the Grinstein ratio of decay constants. It is now possible to determine the relevant quantities on the lattice as accurately as 5% and better,

and the precise determination of $|V_{td}|$ is realized when ΔM_{B_s} , f_{D_d} and f_{D_s} are measured in the forthcoming experiments.

For $|V_{cb}|$, needed for the normalization of the matrix elements, the situation is more promising because the form factor of the $B \rightarrow D^* l \nu$ decay can be determined on the lattice with a few percent accuracy. The lattice method for this calculation is established, and does not receive significant effects from chiral logarithms. It only remains to apply the method to unquenched calculations, and independent checks using other processes are in progress.

The extraction of $|V_{ub}|$ from $B \rightarrow \pi(\rho)$ is still theoretically challenging as it is so in experiment. The form factor calculations still have large uncertainties ($\sim 20\%$) even in the quenched approximation. Further improvements are necessary. In particular, it is essential to find out how one can secure clean signals from simulations and how to overcome the problem of chiral logarithms.

To achieve better accuracy, improvements in the methodology are also crucial. For the issue of chiral logarithm, lowering the quark mass provides the direct route for resolution for which staggered light quarks may be helpful. Clean signals in form factor calculations are obtained with anisotropic lattices. Finally the non-perturbative accuracy might be achieved by the development of the step scaling method.

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